SOME DISTRIBUTIONS ARISING AS A

CONSEQUENCE OF ERRORS IN INSPECTION

by

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ABSTRACT

In this paper the authors survey and consolidate their investigations during the years 1980-1983 dealing with consequences of errors in inspection sampling models. Some indication of the current and future research is given. Selective bibliography is presented.

1. INTRODUCTION

This is a condensed and consolidated account of some results of investigations we have carried out in the last few years ([A]- [G]) on the consequences of errors in inspection sampling. The topic is not new. As can be seen from the list of 'Other References'. It was probably introduced by Lavin in 1946, but did not receive wider attention until the late 1960's. The bulk of the research has been carried out - to the best of our knowledge - since 1979. Our contributions are mainly in the establishment of structures of distributions (of apparent and actual numbers of defective - 'nonconforming' - items) when lot size is finite, for a variety of sampling and measurement situations.

For computational purposes, the structural forms we obtain are very convenient. We will present only a few explicit mathematical formulas, mostly in Section 2, where there is also some background information on the methods of analysis employed. Further details can be found in [A]-[G].

2. MATHEMATICAL NOTATION AND ANALYSIS

We will use

Symbols	to denote	Distributions
Bin(n,p)		$\Pr[X=x]=\binom{n}{x}p^x(1-p)^{n-x}$ $(x=0,1,\ldots,n)$
Hypg(n,D,N)		$\Pr[X=x]=\binom{D}{x}\binom{N-D}{n-x}/\binom{N}{n}$ $(\max(0,n-N+D)\le x\le\min(n,D))$
Mult$(n;\underset{\sim}{p})$ (or Mult$(n;p_1,\ldots,p_k)$)		$\Pr[\underset{\sim}{X}=\underset{\sim}{x}]=\Pr[X_i=x_i\ ,\ i=1,\ldots,k]$ $=n!\prod_{i=1}^{k}(p_i^{x_i}/x_i!)$ $(\sum_1^k p_i=1;\ 0\le x_i;\ \sum x_i=n)$
Mult Hypg$(n;\underset{\sim}{D};N)$		$\Pr[\underset{\sim}{X}=\underset{\sim}{x}]=\left\{\prod_{i=1}^{k}\binom{D_i}{x_i}\right\}\binom{N-\sum D_i}{n-\sum x_i}/\binom{N}{n}$ $(0\le x_i\le D_i;\ n-N+\sum D_i\le\sum x_i\le n)$

We use the symbol to mean 'is distributed as'. Moment calculations are facilitated by using factorial moments

$$\mu_{(s)} = E[X^{(s)}], \text{ or generally } \mu_{(\underset{\sim}{s})} = \mu_{s_1,\ldots,s_k} = E[\prod_{i=1}^{k} X_i^{(s_i)}]$$

where $X^{(s)} = X(X-1)\ldots(X-s+1)$ is the "s-th descending factorial" of X. We have

$$X \frown \mathrm{Bin}(n,p) \quad \Rightarrow \mu_{(s)} = n^{(s)} p^s$$

$$X \frown \mathrm{Mult}(n;p) \quad \Rightarrow \mu_{(\underset{\sim}{s})} = n^{(s)} \prod_{i=1}^{k} p_i^{s_i} \quad (s = \sum_{i=1}^{k} s_i)$$

$$X \frown \mathrm{Hypg}(n;D,N) \quad \Rightarrow \mu_{(\underset{\sim}{s})} = n^{(s)} D^{(s)} / N^{(s)}$$

$$X \frown \mathrm{Mult\ Hypg}(n;\underset{\sim}{D};N) \quad \Rightarrow \mu_{(\underset{\sim}{s})} = n^{(s)} \{ \prod_{i=1}^{k} D_i^{(s_i)} \} / N^{(s)} \quad (s = \sum_{i=1}^{k} s_i)$$

Now consider a lot of N items which is composed of k subsets containing $D_1, D_2, \ldots, D_k$ items $(\sum_{i=1}^{k} D_i = N)$, and suppose that the probability that an item from the i-th subset (which contains D_1 items) is judged to be defective is p_i. What is the distribution of Z, the number of items judged to be defective among those in a random sample of size n chosen (without replacement) from the lot of size N?

If there are $Y_1, Y_2, \ldots, Y_k$ items in the sample from the 1st, 2nd, ..., k-th subset respectively $(\sum_{i=1}^{k} Y_i = n)$ then the corresponding numbers <u>judged</u> to be defective will be conditionally distributed as independent $\mathrm{Bin}(Y_1,p_1)$, $\mathrm{Bin}(Y_2,p_2), \ldots, \mathrm{Bin}(Y_k,p_k)$ variables respectively. Symbolically

$$Z|\underset{\sim}{Y} \frown \mathop{*}_{i=1}^{k} \mathrm{Bin}\ (Y_i, p_i) \tag{1}$$

with * denoting 'convolution'. To obtain the overall distribution of Z, this distribution has to be compounded (mixed) with respect to $\underset{\sim}{Y}$, which has the joint distribution

$$\underset{\sim}{Y} \frown \mathrm{Mult\ Hypg}\ (n;\underset{\sim}{D};N) .$$

(Note that $n-\sum Y_i = N-\sum D_i = 0$ and $\binom{0}{0} = 1$.)

Symbolically

$$Z \sim \left(\mathop{*}_{i=1}^{k} \mathrm{Bin}(Y_i, p_i)\right) \bigwedge_{\underset{\sim}{Y}} \text{Mult Hypg}(n; \underset{\sim}{D}; N) \tag{2}$$

(Λ is the conventional symbol for compounding (mixing).) Conditionally on $\underset{\sim}{Y}$

$$\mu_{(s)}(Z|\underset{\sim}{Y}) = \sum_{j_1} \cdots \sum_{j_k} \binom{s}{j_1 j_2 \ldots j_k} \prod_{i=1}^{k} \left(Y_i^{(j_i)} p_i^{j_i}\right) \tag{3}$$

$$\left(j_i \geq 0; \sum_{i=1}^{k} j_i = s\right)$$

where

$\binom{s}{j_1, j_2 \ldots j_k} = s!/\left(\prod_{i=1}^{k} j_i!\right)$ is a multinomial coefficient; it can conveniently be abbreviated to $\binom{s}{\underset{\sim}{j}}$.

Taking expected values with respect to $\underset{\sim}{Y}$,

$$\mu_{(s)}(Z) = \frac{n^{(s)}}{N^{(s)}} \sum_{j_1} \cdots \sum_{j_k} \binom{s}{\underset{\sim}{j}} \prod_{i=1}^{k} \left(D_i^{(j_i)} p_i^{j_i}\right) \tag{4}$$

From (4)

$$E[Z] = nN^{-1} \sum_{i=1}^{k} D_i p_i = n\overline{p} \tag{5.1}$$

where $\overline{p} = N^{-1} \sum_{i=1}^{k} D_i p_i$ is the probability that an item, chosen at random from the lot, will be classified as 'defective'; and

$$\mathrm{var}(Z) = n \frac{N-n}{N-1} \overline{p}(1-\overline{p}) + \frac{n(n-1)}{N-1} \sum_{i=1}^{k} \frac{D_i}{N} \cdot p_i(1-p_i) \tag{5.2}$$

3. SPECIAL CASES

If there are just two subsets - D defective items and (N-D) non-defectives then

$$E[Z] = n\bar{p} \tag{6.1}$$

$$\text{var}(Z) = n\,\frac{N-n}{N-1}\,\bar{p}(1-\bar{p}) + \frac{n(n-1)}{N}\,\{\frac{D}{N}\,p_1(1-p_1)+(1-\frac{D}{N})p_2(1-p_2)\} \tag{6.2}$$

with $\bar{p} = \frac{D}{N}\,p_1+(1-\frac{D}{N})p_2$. In [B] we give an explicit formula for the distribution of Z and also part of tables of the distribution which we have computed.

If, in this situation, we can assume that there are no false positives (i.e. no 'detection' of nondefectives as 'defective', so that $p_2 = 0$), then

$$\mu_{(s)}(Z) = p_1^{(s)} n^{(s)} D^{(s)} / N^{(s)} \tag{7}$$

whence

$$E[Z] = p_1 nN/D \tag{8.1}$$

$$\text{var}(Z) = p_1^2 n\,\frac{N-n}{N-1} \cdot \frac{D}{N}(1-\frac{D}{N}) + \frac{nD}{N}\,p_1(1-p_1) \tag{8.2}$$

$$= p_1^2 \text{var}(Z|p_1=1) + \frac{nD}{N}\,p_1(1-p_1)$$

(When $p_1 = 1$, Z has a Hypg(n;D;n) distribution.)

4. TWO- AND MULTI-STAGE ACCEPTANCE SAMPLING

A typical two-stage acceptance sampling scheme is summarized below. (Z_j denotes the number of items judged defective in the j-th sample).

Sample	Size	Reject if:	Accept if:	Take next sample if:
First	n_1	$Z_1 > a_1'$	$Z_1 \leq a_1$	$a_1 < Z_1 \leq a_1'$
Second	n_2	$Z_1 + Z_2 > a_2$	$Z_1 + Z_2 \leq a_2$	

All sampling is without replacement.

It is convenient to find the joint distribution of Z_1 and Z_2 without taking into account whether the second sample would be needed under the rules of the sampling scheme. Once the joint distribution is obtained, probabilities (of acceptance, rejection etc) can be computed by summation over relevant values of (Z_1, Z_2).

Denoting by Y_j the actual number of defective items in the j-th sample, we have

$$\Pr[Y_1=y_1;\ Y_2=y_2] = \binom{n_1}{y_1}\binom{n_2}{y_2}\binom{N-n_1-n_2}{D-y_1-y_2}\Big/\binom{N}{D}$$

$$(0 \le y_j < n_j;\ D-N+n_1+n_2 \le y_1+y_2 \le D)$$

- that is

$$\underset{\sim}{Y} = (Y_1,Y_2) \frown \text{Mult.Hypg. } (D;n_1,n_2;N) \tag{9}$$

Conditionally on $\underset{\sim}{Y}$

$$Z_j|\underset{\sim}{Y} \frown \text{Bin}(Y_j,p_1) * \text{Bin}(n_j-Y_j,p_2) \tag{10}$$

and the Z's are mutually independent (j=1,2).

The (unconditional) distribution of $\underset{\sim}{Z} = (Z_1,Z_2)$ is obtained by compounding (10) with (9). Table 1 (from (D)) shows some results of calculations based on this analysis.

The analysis extends in a straightforward way to m-stage sampling schemes (m > 2). We now have

$$\underset{\sim}{Y} = (Y_1,\ldots,Y_m) \frown \text{Mult.Hypg. } (D;\underset{\sim}{n};N) \tag{11}$$

while (10) holds for j=1,...,m.

The structure of the linear regression equation

$$E[Z_i|Z_j] = n_i\bar{p}_i - n_i(p_{i1}-p_{i2})(p_{j1}-p_{j2})\frac{DN^{-1}(1-DN^{-1})}{(N-n_j)\bar{p}_j(1-\bar{p}_j)}(Z_j-n_j\bar{p}_j) \tag{12}$$

(where $\bar{p}_j = DN^{-1}p_{j1}+(1-DN^{-1})p_{j2}$ and $p_{h1}(p_{h2})$ denotes the probability that a defective (nondefective) item will be classified as 'defective' at the h-th stage) is of some interest.

5. MULTIPLE TYPES OF DEFECT

We suppose a random sample of size n taken (without replacement) from a lot of size N wherein there are D items with g_1 defects of type (1) and g_2 of type (2) (with each g_1,g_2, g_j = 0 or 1). For example, there are D_{00}

items with neither type of defect.

In [F] we discussed situations in which inspection is on only one type - (1), say - of defect, and derived the distribution of the number $Z^*_{2(1)}$ of items with defect type (2) among the Z_1 alleged to have defect type (1), as a result of inspection of the sample. Denoting by $Y_{g_1g_2}$, the number of items actually having 'defect pattern' (g_1,g_2) in the sample, the joint distribution of $\underset{\sim}{Y} = (Y_{00},Y_{01},Y_{10},Y_{11})$ is

$$\underset{\sim}{Y} \frown \text{Mult Hypg}(n;\underset{\sim}{D};N) \tag{13}$$

Conditionally on $\underset{\sim}{Y}$ we have

$$Z_1 = W_{00}+W_{01}+W_{10}+W_{11} \tag{14.1}$$

$$Z^*_{2(1)} = W_{01}+W_{11} \tag{14.2}$$

and also

Z^*_1 (number of actual defectives among the Z_1 alleged to be defective)

$$= W_{10}+W_{11} \tag{14.3}$$

where the $W_{g_1g_2}$'s are mutually independent and

$$W_{g_1g_2} \frown \text{Bin}(Y_{g_1g_2}, g_1p_1 + (1-g_1)p_2) \tag{15}$$

$W_{g_1g_2}$ corresponds to the contribution to Z_1 from the $Y_{g_1g_2}$ members of the sample with actual defect pattern (g_1,g_2).

Generalization to m types of defect and 'defect patterns' $(\underset{\sim}{g}) = (g_1,g_2,\ldots,g_m)$ is straightforward.

In [G] we considered situations in which each item in the sample is tested for m ($\geq$ 2) types of defect. We now have $Z_{\underset{\sim}{g}}$ denoting the number of items observed to have defect pattern $(\underset{\sim}{g})$, among which the number actually having defect pattern $(\underset{\sim}{h})$ is denoted by $Z^*_{\underset{\sim}{h}(\underset{\sim}{g})}$.

Denoting by $p_{i1}(p_{i2})$ the probability that a defect of type (i) will be 'detected' when in reality it is (is not) present, the probability that an actual defect pattern $(\mathbf{g})$ will be classified as $(\mathbf{h})$ is

$$P_{\mathbf{g}|\mathbf{h}} = \prod_{i=1}^{m} \{p_{i1}^{g_i h_i} p_{i2}^{g_i(1-h_i)} (i-p_{i1})^{(1-g_i)h_i} (1-p_{i2})^{(1-g_i)(1-h_i)}\} \tag{16}$$

and $$Z_{\mathbf{g}} = \textstyle\sum_{\mathbf{h}} W_{\mathbf{g}|\mathbf{h}} \tag{17.1}$$

$$Z^*_{\mathbf{h}}(\mathbf{g}) = W_{\mathbf{g}|\mathbf{h}} \tag{17.2}$$

where the $W_{\mathbf{g}|\mathbf{h}}$'s are independent and

$$W_{\mathbf{g}|\mathbf{h}} \frown \operatorname{Bin}(Y_{\mathbf{h}}, P_{\mathbf{g}|\mathbf{h}})$$

The 2^m $Y_{\mathbf{h}}$'s (actual number of items with defect pattern (h) in the sample) have the joint distribution

$$\mathbf{Y} \frown \text{Mult Hypg}(n;\mathbf{D},N) \tag{18}$$

Unconditional distributions are obtained by compounding (17) with regard to the distribution (18) of $\mathbf{Y}$.

We note that (using Bayes' formula) the probability that an item classified as having defect pattern $(\mathbf{g})$, as a result of inspection, actually has pattern $(\mathbf{h})$ is

$$p_{\mathbf{h}(\mathbf{g})} = P_{\mathbf{h}} P_{\mathbf{g}|\mathbf{h}} / \overline{P}_{\mathbf{h}} \tag{19}$$

where, for an item chosen at random from the lot

$$P_{\mathbf{h}} = \Pr[\text{defect pattern } (\mathbf{h})] = D_{\mathbf{h}}/N \tag{20}$$

and

$$\overline{P}_{\mathbf{h}} = \Pr[\text{classified as pattern } (\mathbf{h}) \text{ after inspection}]$$

$$= \textstyle\sum_{\mathbf{f}} P_{\mathbf{f}} P_{\mathbf{h}|\mathbf{f}} = \sum_{\mathbf{f}} P_{\mathbf{h}|\mathbf{f}} D_{\mathbf{f}}/N \, . \tag{21}$$

6. SCREENING AND HIERARCHAL SCREENING

A somewhat different situation arises in connection with certain screening techniques, appropriate (for example) to detection of *contaminant or* critical impurity in a liquid product. Here there is no 'sampling element as such except in the selection of n containers from a lot of N containers. If each of n containers are tested separately, n tests are needed, but if material from each of the n lots is mixed and tested, only one test is needed if a negative result is obtained on testing the mixture. If a positive result is obtained each lot is tested separately, so (n+1) tests in all are needed. It is to be expected that if the proportion (D/N) of containers with impurities is small, the expected (average) number of tests needed will be less than n.

Let $p_1'(p_2')$ denote the probability of 'detecting' the impurity when it is (is not) really present in the mixture. The overall probability of obtaining a positive result on testing the mixture from the n containers is

$$\{1-P_0(n)\}p_1'+P_0(n)p_2' = p_1' - (p_1'-p_2')P_0(n) \tag{22}$$

where $P_0(n) = (N-D)^{(n)}/N^{(n)}$ is the probability that none of the n containers have the impurity. The expected number of tests is therefore

$$1 + n\{p_1 - (p_1-p_2)P_0(n)\} \tag{23}$$

The probability of correct classification for defective items is $p_1'p_1$ where (as before) p_1 is the probability of detection where containers are inspected singly. The probability of correct classification for nondefective items is

$$P_0^*(n)(1-p_2'p_2) + (1-P_0^*(n))(1-p_1p_2) \tag{24}$$

where $P_0^*(n) = (N-D-1)^{(n-1)}/(N-1)^{(n-1)}$ is the conditional probability that none of the n containers have the impurity, given that one does not. (22), (23) and (24) are the essential indices for assessing the effectiveness of the screening plan.

Similar calculations can be made for hierarchal screening (see [C]). Our formula allow for classification probabilities to vary with screening stage, but not otherwise. It may well be, of course, that in reality, the probability of detecting impurity in a mixture increases with the number of containers in which the impurity is present.

7. DETECTION OF FAULTY INSPECTION

A brief initial study of problems arising in trying to detect the existence of errors in inspection is given in [E]. If N is large, it is impossible to detect such errors simply for records of the result of inspection since for such cases the distribution of Z is a binomial (or convolution of binomials) with parameter(s) in which $\underset{\sim}{D}$ (numbers of defects in the lot) and $\underset{\sim}{p}$ (probabilities of 'detection' of defect) are compounded.

If N is not too large (n/N not too small) it is, in principle, possible to test for errors in inspection. An indication of the way in which p_2 (probability of false position) is known to be zero - so one is simply testing the hypothesis $p_1 = 1$ - is given in (E). However, it is clear that the sensitivity of such a test will be rather weak, unless the sampling function (n/N) is quite large and a considerable number of absent Z's obtained under the same conditions is available.

8. FUTURE RESEARCH

We are presently engaged in extension of the work described in Section 5 when there is some form of structure in the types of defect. For example, the m types of defects might be classified into r groups of $m_1, m_2, \ldots, m_r$ types ($m_1 + \ldots + m_r = m$), and acceptance require no defect in any one of the r groups. We will inter alia investigate the applicability of some coding theory concepts and results in this context.

Explicit introduction of cost and loss functions is also under consideration.

Table 1: Acceptance Probabilities

$n_1 = n_2 = 13; \; a_1 = 0, \; a_1' = a_2 = 2$

P_1	P_2=0	0.01	0.02	0.05	0.10	0	0.01	0.02	0.05	0.10
	N = 100, D = 5					N = 200, D = 10				
1.00	0.9021	0.8374	0.7678	0.5565	0.2783	0.8862	0.8263	0.7611	0.5586	0.2838
0.98	0.9071	0.8435	0.7747	0.5638	0.2834	0.8915	0.8325	0.7679	0.5656	0.2886
0.95	0.9143	0.8525	0.7848	0.5747	0.2910	0.8993	0.8417	0.7780	0.5761	0.2959
0.90	0.9257	0.8669	0.8013	0.5929	0.3040	0.9116	0.8565	0.7945	0.5937	0.3083
0.75	0.9543	0.9055	0.8474	0.6470	0.3446	0.9437	0.8971	0.8413	0.6466	0.3473
	N = 100, D = 10					N = 200, D = 20				
1.00	0.5292	0.4661	0.4081	0.2655	0.1191	0.5374	0.4768	0.4202	0.2778	0.1268
0.98	0.5446	0.4807	0.4217	0.2757	0.1244	0.5517	0.4903	0.4329	0.2874	0.1320
0.95	0.5679	0.5029	0.4424	0.2914	0.1327	0.5733	0.5110	0.4523	0.3023	0.1400
0.90	0.6069	0.5402	0.4777	0.3186	0.1475	0.6096	0.5459	0.4854	0.3282	0.1542
0.75	0.7218	0.6537	0.5872	0.4080	0.1987	0.7184	0.6535	0.5894	0.4137	0.2036
	N = 100, D = 20					N = 200, D = 40				
1.00	0.0830	0.0709	0.0605	0.0372	0.0162	0.0975	0.0836	0.0714	0.0442	0.0192
0.98	0.0912	0.0781	0.0667	0.0412	0.0180	0.1060	0.0909	0.0779	0.0483	.0211
0.95	0.1048	0.0899	0.0769	0.0477	0.0209	0.1198	0.1030	0.0884	0.0552	.0242
0.90	0.1311	0.1128	0.0968	0.0606	0.0269	0.1463	0.1263	0.1087	0.0684	0.0304
0.75	0.2413	0.2105	0.1830	0.1183	0.0547	0.2547	0.2228	0.1943	0.1264	0.0586

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error in inspection sampling; discrete distributions; acceptance sampling; screening; detection of faulty inspection

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper the authors survey and consolidate their investigations during the years 1980-1983 dealing with consequences of errors in inspection sampling models. Some indication of the current and future research is given. Selective bibliography is presented.